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TURBULENT THERMAL BOUNDARY LAYER ON A FLAT PLATE WITH A THERMALLY INSULATED SECTION

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The influence of an initial adiabatic section on the heat transfer coefficient is studied on the assumption that the heat transfer law is expressed in the form $\mathrm{St} = A/\mathrm{Re}_T^{**m}\,\mathrm{Pr}^n$. In addition, the temperature distribution on a thermally insulated wall is obtained when heat transfer occurs over the initial section.

A great deal of attention has recently been given to investigating the influence of upstream conditions on the development of a turbulent boundary layer [1-4].

We shall examine the case of a turbulent boundary layer on a flat plate with an initial thermally insulated section, when heat transfer at $x > x_0$ satisfies the condition t_W = const (Fig. 1a).

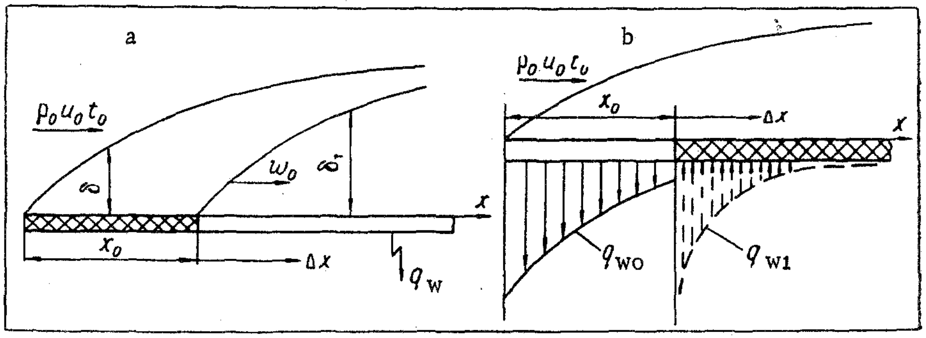

Fig. 1. Plates with initial insulated section (a) and with initial heat transfer section (b).

The energy equation for the thermal boundary layer, taking into account that the velocity may vary at its outer edge, is written as:

$$\frac{d\,[\delta_T^{**} w_0]}{dx\, w_0} = \frac{\alpha}{g\rho_0 w_0 c_{p0}} = \mathrm{St}_1. \tag{1}$$

Equation (1) is similar in form to the energy equation in the presence of a longitudinal pressure gradient. However, a longitudinal pressure gradient has no appreciable influence on heat transfer [1].

For a turbulent boundary layer

$$\mathrm{St}_1 = \frac{\alpha}{g\rho_0 w_0 c_{p0}} = \frac{A}{\mathrm{Re}_{T_1}^{**0.25}\mathrm{Pr}^{0.75}}, \tag{2}$$

where St_1 is the Stanton number based on the velocity at the edge of the thermal boundary layer, and $\mathrm{Re}_{T_1}^{**} = w_0\delta_T^{**}/\nu$. Assuming a power-law velocity profile

$$w_0 = u_0\left(\delta_T/\delta\right)^{1/7} = u_0\left(\delta_T^{**}/\delta^{**}\right)^{1/7}, \tag{3}$$

then

$$\mathrm{Re}_{T_1}^{**} = w_0\delta_T^{**}/\nu = (u_0\delta^{**}/\nu)\,(\delta_T^{**}/\delta^{**})^{8/7}. \tag{4}$$

From (2), (3), and (4) we obtain

$$\mathrm{St} = \alpha/g\rho_0 u_0 c_{p0} = (A/\mathrm{Re}^{**0.25}\mathrm{Pr}^{0.75})\,(\delta/\delta_T)^{1/7}, \tag{5}$$

where St is the Stanton number based on the free-stream velocity, and $\mathrm{Re}^{**} = u_0\delta^{**}/\nu$ is the Reynolds number based on the free-stream velocity and momentum thickness. Taking into account that

$$\frac{Cf_0}{2} = A/\mathrm{Re}^{**\,0.25} = \mathrm{St}_0\,\mathrm{Pr}^{0.6}, \tag{6}$$

we obtain

$$\frac{\mathrm{St}}{\mathrm{St}_0} = \left(\frac{\delta^{**}}{\delta_T^{**}}\right)^{1/7} \frac{1}{\mathrm{Pr}^{0.15}} = \left(\frac{\delta_x^{**}}{\delta_{\Delta x}^{**}}\right)^{1/7}. \tag{7}$$

The values δ_x^{**} and $\delta_{\Delta x}^{**}$ are found from solution of the equations

$$\mathrm{Re}_x^{**} = (0.016\mathrm{Re}_x)^{0.8}, \quad \mathrm{Re}_{\Delta x}^{**} = (0.016\mathrm{Re}_{\Delta x})^{0.8}. \tag{8}$$

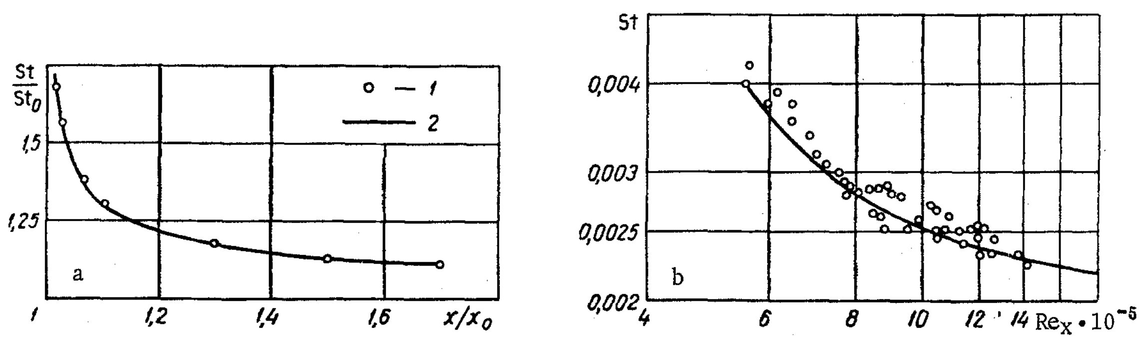

Fig. 2. Effect of initial, thermally insulated section on the heat transfer coefficient: a) 1 – according to (10); 2 – according to (9); b) 1 – experiment [4]; 2 – according to (11).

Finally, the function giving the effect of the initial adiabatic section on the heat transfer coefficient is

$$\varphi(x, x_0) = \mathrm{St}/\mathrm{St}_0 = [x/(x - x_0)]^{0.114}. \tag{9}$$

It can be seen from Fig. 2a that the calculation based on this equation agrees well with the equation proposed by Seban, and this gives a good description of the experimental results of Reynolds, Kays, and Kline [3]:

$$\mathrm{St}/\mathrm{St}_0 = [1 - (x_0/x)^{0.9}]^{-1/9} \tag{10}$$

The plate formula (10), obtained using the conservation of heat law with variable velocity at the edge of the thermal boundary layer, coincides with that obtained in [1] for the same case using Prandtl's hypothesis. We may write (9) in the form

$$\mathrm{St} = 0.029\,(u_0 x/\nu)^{-0.2}\,\mathrm{Pr}^{-0.6}\,[x/(x - x_0)]^{0.114}. \tag{11}$$

Figure 2b shows calculations according to this formula compared with experimental results for a flat plate under the condition q_w = const [4]. It is known that the heat transfer coefficients for constant heat flux and constant wall temperature are practically the same.

We shall now examine the case of a turbulent boundary layer on a flat plate with an initial heat transfer section followed by a thermally insulated section, with heat transfer occurring at $x < x_0$ under the condition of constant wall temperature (Fig. 1b). The temperature of the thermally insulated wall ($x > x_0$) has to be determined.

We will further determine the heat flux q_{w1}, which must be supplied in the section $x > x_0$ in order that the wall temperature for $x > x_0$ be equal to the free-stream temperature (Fig. 1b).

The energy equation for an incompressible boundary layer is linear in temperature, and therefore, applying the method of superposition, we may write an expression for the heat flux [2] as follows:

$$q_{w1} = g\rho_0 w_0 c_{p0} [\mathrm{St}_0 (t_{w0} - t_0) + \mathrm{St}_0 \varphi(x, x_0)(t_0 - t_{w0})]. \tag{12}$$

The same heat flux q_{w1} may, however, be written in another way:

$$q_{w1} = \alpha (t_0 - t_{adw}). \tag{13}$$

In this case, using the heat transfer law in the form (2), we obtain

$$q_{w1} = Ag\rho_0 u_0 c_{p0} (t_0 - t_{adw}) / \mathrm{Re}_{T_1}^{**m} \mathrm{Pr}^n; \tag{14}$$

when $x \to \infty$ we have

$$\int_{x_0}^{x} q_{w1}\, dx = \int_{0}^{x} q_{w0}\, dx. \tag{15}$$

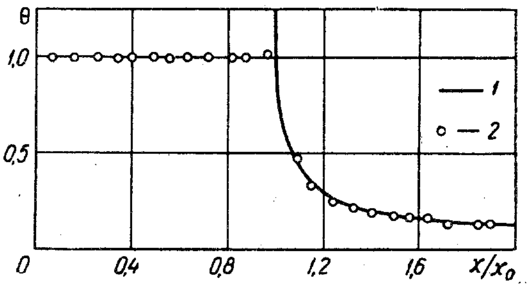

Fig. 3. Surface temperature distribution: 1) theory from (20); 2) experiment from [3].

It is evident that the extension of this condition to finite values of x will not introduce an appreciable error, since q_{w1} is exponential in nature, and there will be a noticeable change of $\int_{x_0}^{x} q_{w1}\, dx$ only for small values of x.

From (15) we find that

$$\mathrm{Re}_{T_1}^{**} = \mathrm{Re}_{T_0}^{**} / \Theta. \tag{16}$$

In view of this, (15) is transformed to

$$q_{w1} = Ag\rho_0 u_0 c_{p0} (t_0 - t_{adw})\, \Theta^m / \mathrm{Re}_{T_0}^{**m} \mathrm{Pr}^n. \tag{17}$$

From (12) and (17), the dimensionless wall temperature is

$$\Theta = \frac{t_0 - t_{adw}}{t_0 - t_{w0}} = [\varphi(x, x_0) - 1]^{\frac{1}{m+1}} \left(\frac{\mathrm{Re}_{T_0}^{**}}{\mathrm{Re}_T^{**}} \right)^{\frac{m}{m+1}}. \tag{18}$$

For a turbulent boundary layer the exponents in the heat transfer law are m = 0.25; n = 0.75. Function $\varphi(x, x_0)$ is taken in the form (9) and $\mathrm{Re}_{T_0}^{**}$ and Re_T^{**} are found from the solution of the energy equation in integral form, using the heat transfer relation in the form (2); they are equal, respectively, to:

$$\mathrm{Re}_{T_0}^{**} = \left[A(m+1) \frac{1}{\mathrm{Pr}^n} \mathrm{Re}_{x_0} \right]^{0.8}, \quad \mathrm{Re}_T^{**} = \left[A(m+1) \frac{1}{\mathrm{Pr}^n} \mathrm{Re}_x \right]^{0.8}. \tag{19}$$

In accordance with the relation given above for a turbulent boundary layer, (18) may be written as

$$\Theta = \left[\left(\frac{x}{x - x_0} \right)^{0.114} - 1 \right]^{0.8} \left(\frac{x_0}{x} \right)^{0.16}. \tag{20}$$

Calculations based on (20) and the experimental data of [3] are compared in Fig. 3.

NOTATION

x_0 – length of initial section; t_0 and t_w – temperature of the gas outside boundary layer and wall temperature; t_{adw} – adiabatic wall temperature; t_{w_0} – wall temperature in heat transfer section ρ_0 – density of gas outside boundary layer; c_{p0} – specific heat of gas outside boundary layer; μ_0 – dynamic viscosity of gas outside boundary layer; λ_0 – thermal conductivity of gas outside boundary layer; u_0 – velocity outside boundary layer; w_0 – velocity at edge of thermal boundary layer; δ and δ_T, δ_T^{**} and δ^{**}, $\delta_{T_0}^{**}$ – dynamic and thermal boundary layer thicknesses, convection and momentum thicknesses, and convection thickness for $x = x_0$; $\delta_{x,\Delta x}$ – dynamic boundary layer thicknesses during development from points $x = 0$ and $x = x_0$, respectively; $\delta_{x,\Delta x}^{**}$ – momentum thicknesses during development of (dynamic) boundary layer from points $x = 0$ and $x = x_0$, respectively; q_w, q_{w0}, q_{w1} – heat fluxes (Fig. 1) α – heat transfer coefficient $\Delta x = x - x_0$; $\Theta = (t_0 - t_{ad.w})/(t_0 - t_{w0})$; $Re_x = \rho_0 u_0 x/\mu_0$; $Re_{\Delta x} = \rho_0 u_0 \Delta x/\mu_0$; $Re^{**} = \rho_0 u_0 \delta^{**}/\mu_0$; $Re_x^{**} = \rho_0 u_0 \delta_x^{**}/\mu_0$; $Re_{\Delta x}^{**} = \rho_0 u_0 \delta_{\Delta x}^{**}/\mu_0$; $Re_{T_0}^{**} = \rho_0 u_0 \delta_{T_0}/\mu_0$; $Re_T^{**} = \rho_0 u_0 \delta_T^{**}/\mu_0$; $Pr = c_{p0}\mu_0/\lambda_0$; $St = \alpha/g\rho_0 u_0 c_{p0}$; $St_1 = \alpha/g\rho_0 w_0 c_{p0}$; St_0 – Stanton number on flat plate, when dynamic and thermal layers start at same point; Cf_0 – friction coefficient on flat plate; A, m, n – constants in the heat transfer relation.

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